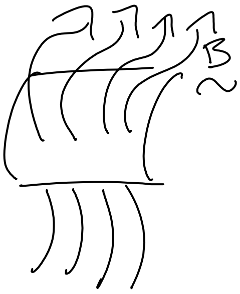
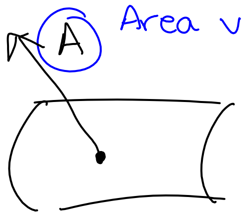
# Lecture 2

Tuesday, 11 August 2009
4:35 PM

## Magnetic Flux

The magnetic flux crossing a particular surface can be calculated from




Area vector $\perp$ to area

$$\phi = \int \underline{B} \cdot dA$$

Unit of flux is the weber and so 1 Tesla = 1 weber/m$^2$

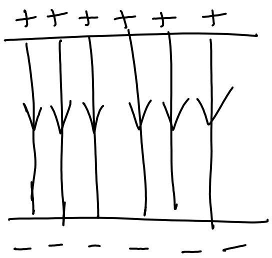
Note that because flux is $\int \underline{B} \cdot dA$ then flux is a scalar quantity.

Differentiating

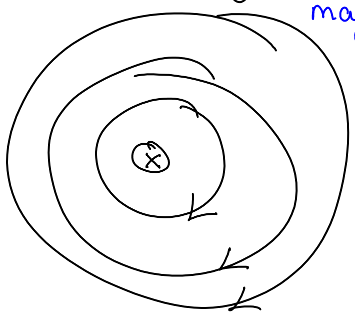
$$B = \frac{d\phi}{dA}$$

Both electric and magnetic flux are visualized by "lines" as an aid of thought.

electric flux




magnetic flux



closed surface integral

dot product

$$\oint \underline{B} \cdot d\underline{A} = 0$$

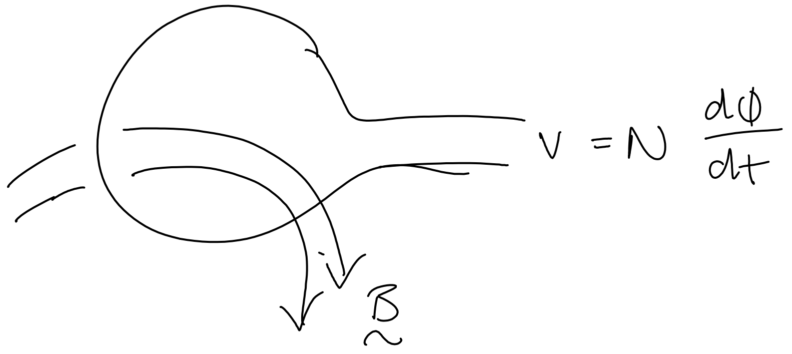
only holds for magnetic flux

$$\oint E \cdot ds = \frac{\Phi}{\varepsilon}$$ = electric charge

describes the continuous nature of B

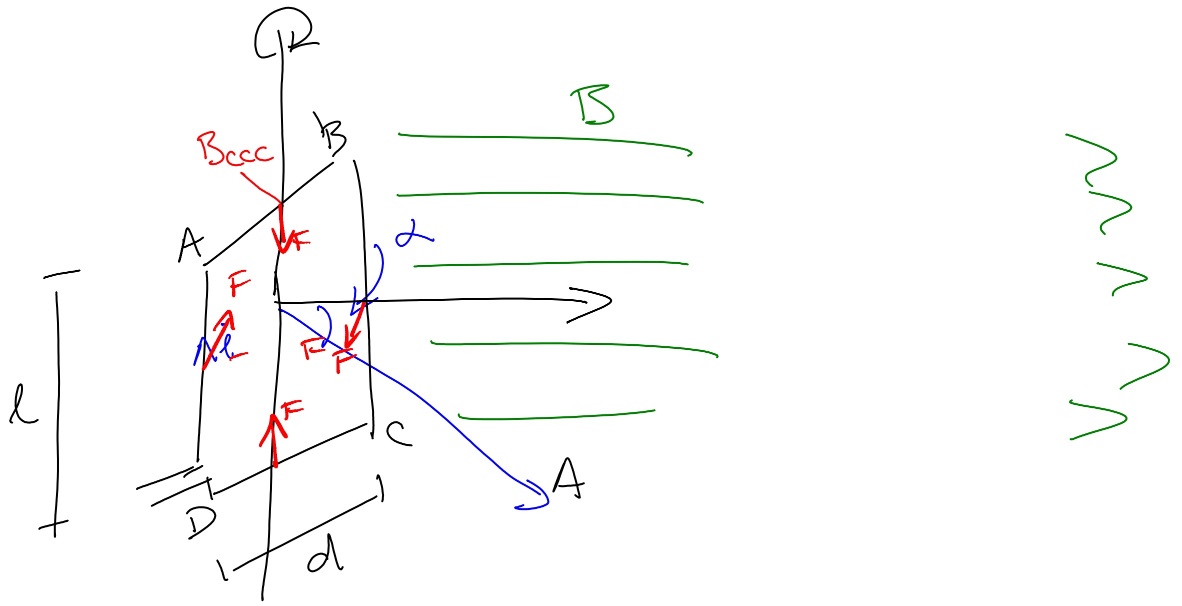
· describes the continuous nature of $\underline{B}$

· There is no magnetic charge.

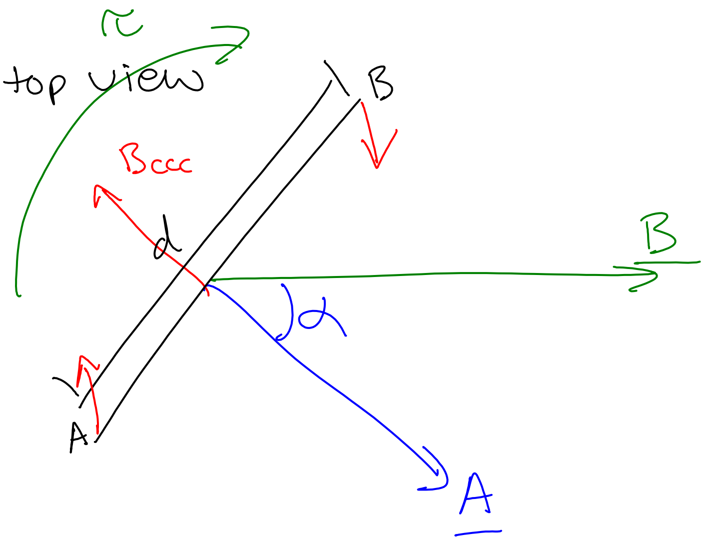



## Magnetic Moment

Consider a CCC in a magnetic field.




top view



The forces on AB & CD are equal and opposite and so cancel.

The forces on BC and DA cause a torque on the loop.

$$F = BIl \quad \because \quad \underline{B} \perp \underline{l}$$

$$\text{Torque} = 2F\frac{d}{2}\cos\left(\frac{\pi}{2} - \alpha\right)$$

$$= 2BIl\frac{d}{2}\sin\alpha$$

$$= BIld \sin\alpha$$

$$= BI(A)\sin\alpha$$

$$\boxed{\text{Torque} = BI(A)\sin\alpha}$$

The product IA is referred to as the magnetic moment of the loop (Unit $= Am^2$)

let 'm' be magnetic moment (vector)

$$\therefore T = mB\sin\alpha$$

$$\iff \underline{\tau} = \underline{m} \times \underline{B}$$

Note that torque will tend to align the magnetic field of the current ($B_{cc}$) the external magnetic field ($B_{ext}$)

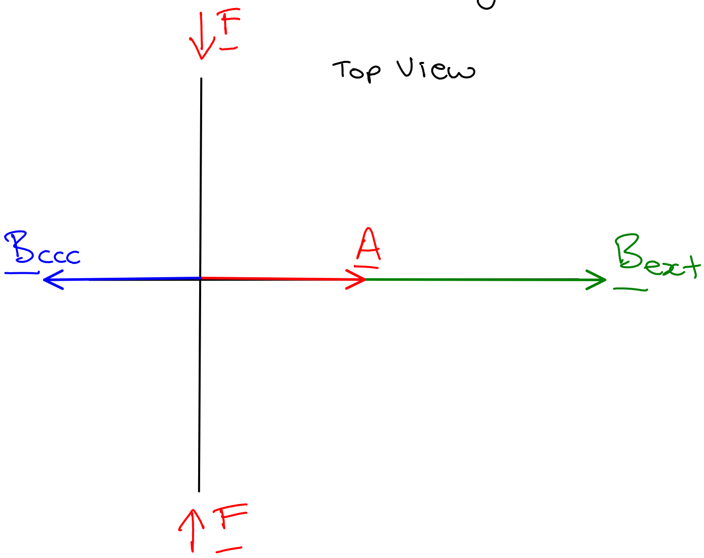
For the case where $B_{ext}$ is not uniform

$$\delta \underline{T} = \delta \underline{m} \times \underline{B}$$

may be used to evaluate the torque.

## Alignment of the magnetic moment

Consider the loop again at an angle of $\alpha = 0$



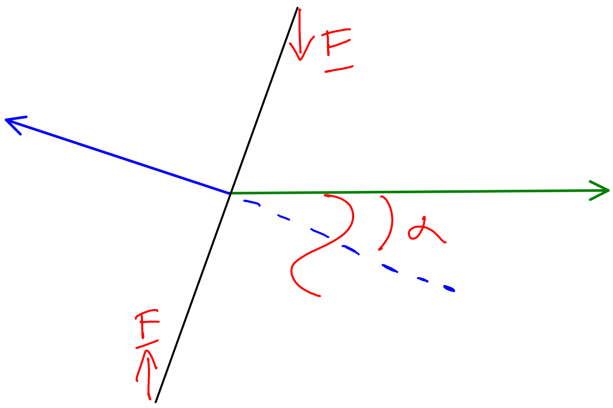

$$\underline{\tau} = \underline{m} \times \underline{B}$$

$$= IAB \sin \alpha$$

$$= 0$$

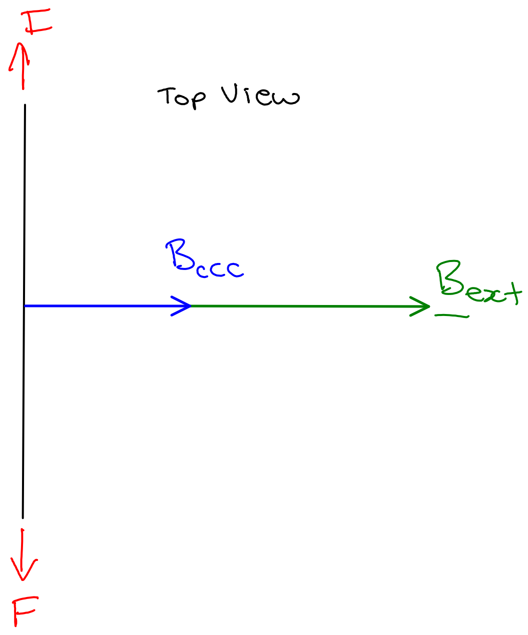
$\therefore$ loop is in equilibrium but it is an unstable equilibrium.

Slight Perturbation:




$$\underline{\tau} = \underline{m} \times \underline{B} \neq 0$$

This will result in a torque and a rotation of the ccc until of the CCC until '




$$\underline{\tau} = \underline{m} \times \underline{B} = 0$$

Again, the loop is in equilibrium, but this time it is a stable equilibrium.

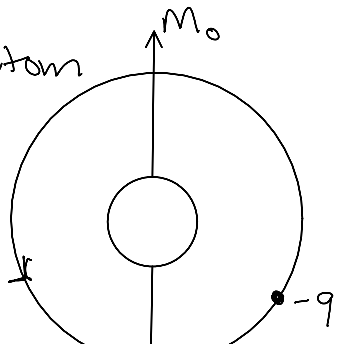
Conclusion:

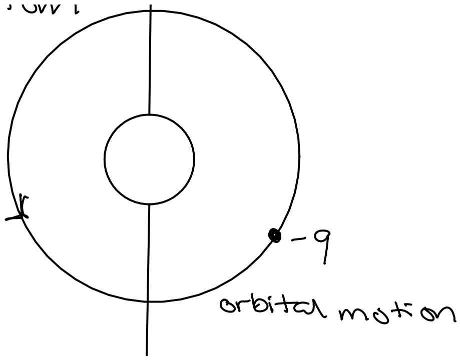
If $\underline{B}_{ext}$ and $\underline{B}_{ccc}$ are not aligned, a torque will act on the CCC which tends to align $\underline{B}_{ccc}$ and $\underline{B}_{ext}$. When aligned, the equilibrium is stable.

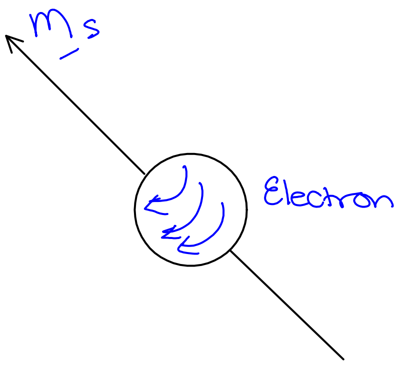
This result can be extended to a general magnetic moment in an external magnetic field. A torque will act to align the magnetic moment $\underline{m}$ with the external magnetic field.

## Magnetic Materials

Consider an atom



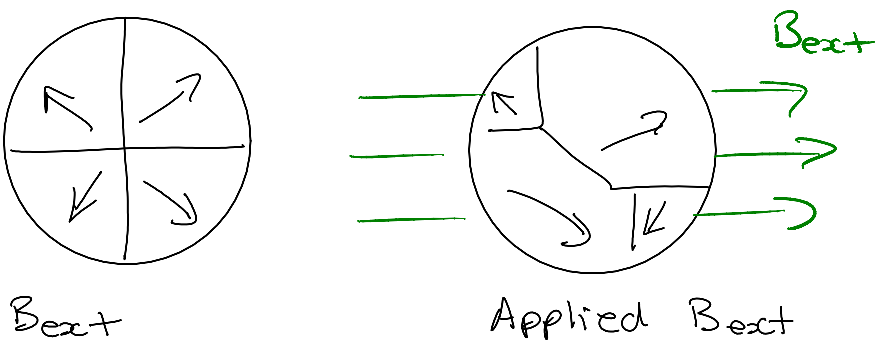


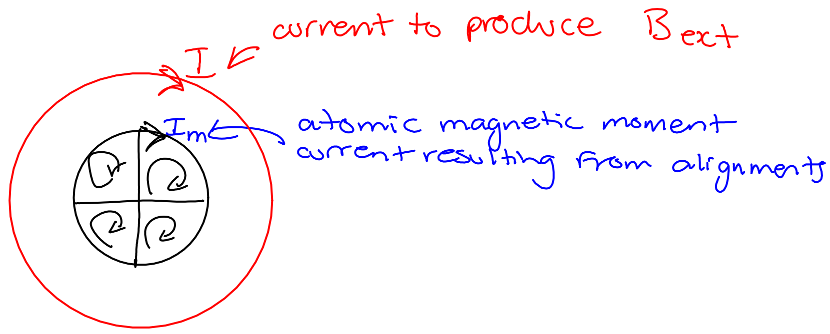

In ferromagnetic materials the atoms are arranged so that thier magnetic moments $\underline{m}$ add rather than subtract.

At room temperature Fe, Ni & Co are ferromagnetic

Experimentally it has been shown that ferromagnetic materials are divided into domains. When a specimen of ferromagnetic material is placed in a magnetic field, the m of the atoms tend to rotate into alignment with $\underline{B}_{ext}$

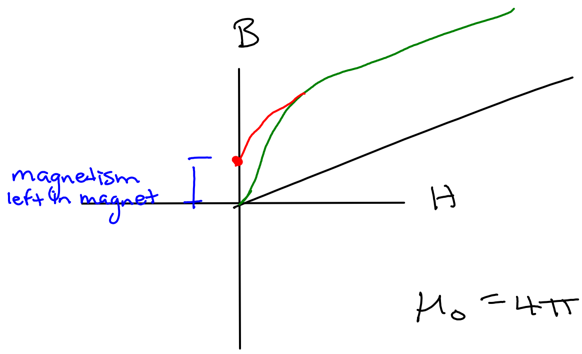



## Magnetic Field Inside a Material Medium




Note that $I_m$ tends to increase $I$, which implies that $\underline{B}$ is increased inside the medium,

$$B = \mu_0 H$$




$$\mu_0 = 4\pi \times 10^{-7} \quad H/m$$